Root locus analysis –
An excellent tool for rotating machine design and analysis

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The potential for unstable operation is inherent in all feedback control systems. Consequently, it is desirable to analyze their stability characteristics and their damping characteristics so that both the absolute stability and the relative stability (peak overshoot, settling time, etc.) of the system can be determined. Classical control theory uses a method commonly known as root locus analysis, developed by W. R. Evans [1] in 1948, to accomplish this.

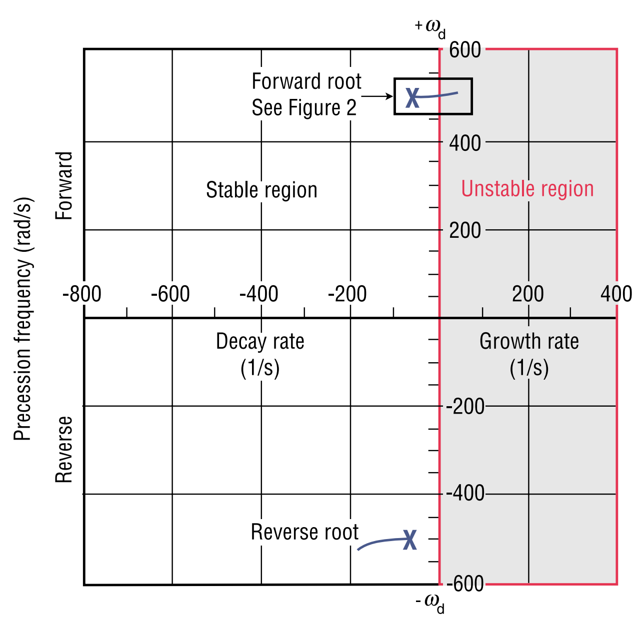
All rotating machines can be modeled as closed loop control systems, where the machine responds characteristically to forces and moments. This allows them to be studied using control theory techniques. Historically, graphical tools, such as Campbell diagrams and logarithmic decrement plots, were used to determine a system's natural frequencies and damping factors. These methods can be cumbersome and cannot provide a method of analysis. However, design and analysis can be obtained from a graphical representation of the root locus.

To understand some of the features of root locus analysis, consider a simple model for a rotor supported in a fluid-film bearing. The equation for the characteristic response of a rotor/bearing/seal system is:

$$Ms^2+Ds+K-jD\lambda\Omega = 0$$

Where M, D, and K are the lumped generalized first lateral mode inertia (lb·sec²/in), damping (lb·sec/in), and stiffness (lb/in), which encompass the bearing's hydrodynamic and hydrostatic (externally pressurized) fluid-film direct stiffness, and the shaft radial stiffness, λ, is the fluid circumferential average velocity ratio (dimensionless), and Ω is the rotor rotative speed (rad/s).

The roots of the characteristic equation are determined by treating it as a quadratic equation. The rotor speed is incremented from zero, and the roots calculated and plotted. Any parameter can be iterated in this manner to determine the sensitivity of the system to variations of that parameter. Since this is a second order system, two complex roots will exist (Figure 1), that move in opposite directions as rotative speed increases. In the plot, the positive portion of the vertical axis is the forward precession (in the same direction as the rotor rotation) rate of the rotor response, and the negative portion is the reverse precession (opposite rotor rotation direction) rate. The horizontal axis is the rate at which the amplitude of the shaft motion decreases or increases. The



In this graph:

1) The variable parameter is rotor rotative speed, Ω

2) There are two root loci: one for forward precession ($+\omega_d$), one for reverse precession ($-\omega_d$).

3) The forward root becomes unstable at Ω =1050 rad/s.

4) The reverse root poses no threat to stability because it moves toward greater stability.

5) "X" indicates Ω=0

Figure 1. Root locus plot for a simple rotor system.

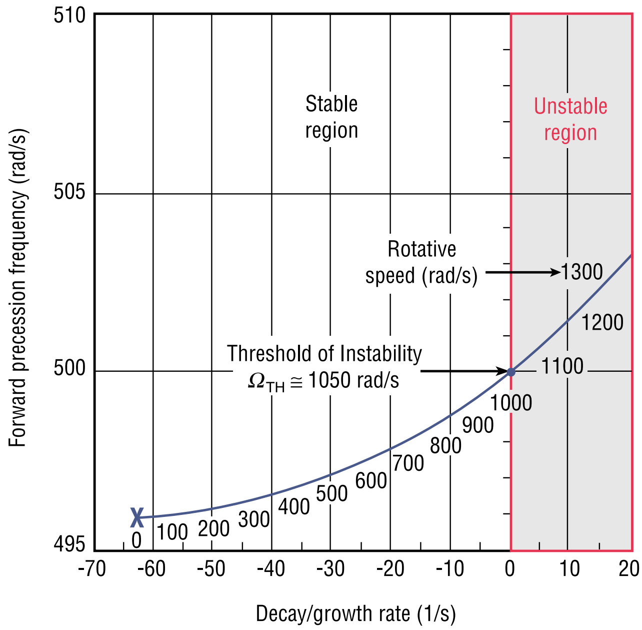


Figure 2. The forward root of a simple rotor system.

further left on the plot that the root exists, the more damping exists to control the decay rate of the rotor motion. When the root encounters the vertical axis (at Ω = 1050 rad/s, Figure 2), the system is marginally stable and the rotor transient motion will neither grow nor decay. This is the Threshold of Instability, and the rotor system will be unstable for rotating speeds above this threshold speed.

The root of interest for absolute stability is the forward precession (above the horizontal axis) root, because the reverse root moves toward greater stability. Figure 2 shows the forward root locus in greater detail. Note that the rotor speed for the Threshold of Instability is 1050 rad/s. However, the precession rate of the motion at that point is 500 rad/s, less than half the rotative speed, which is typical for fluid journal bearings. Therefore, this machine, as described by the model, should not be operated above 1050 rad/s. As the machine stiffness changes, due to changes in operating condition, this stability condition will vary. To study this, families of the locus can be generated for variations in stiffness, damping, and mass to determine the most stable operating regions.

This brief example demonstrates just a few of the features of the root locus technique. Please refer to reference 2 for a more extensive treatment of typical control system design and analysis techniques.

References

1. Evans, W. R., *Control-system Dynamics*, pp.117, McGraw-Hill Book Company, Inc., New York, New York, 1954.
2. Ogata, Katsuhiko, *Modern Control Engineering*, Third Edition, Prentice-Hall, Inc., 1997.