Root Locus Made Easy (Really!)

Stop! Don't turn this page! If the title "Root Locus" conjures gardening images or, worse, makes you want to quickly move onto the next article, then keep reading. Hopefully, this article will clarify a concept that is very important to Bently Pressurized Bearing Co. because it has such practical application to stability problems in rotating machinery.

Stable or Unstable? That is the Question

Simply put, root locus is a method that allows us to characterize the *stability* of a system. We don't think much about stability until things go, well, unstable. For instance, many of us watch the stock market with great personal interest. While the stock market frequently dips in response to bad news, it usually recovers much to our relief and returns to its original level and hopefully, even keeps growing. Although one might reasonably argue the fact, we can say the stock market is *stable* because it basically **returns to its original state of equilibrium following brief disturbances** such as poor earnings results and other bad economic news.

We don't have to look too far back to a time when the stock market was clearly *unstable*. Conditions leading up to the Great Depression set the stage for the panic on "Black Tuesday" 29 October 1929. In one day, American common stocks dropped approximately 12% in value. There have been larger single-day drops such as on 19 October 1987, however, the market continued to lose 40% of its value by the end of 1929 because economic conditions in the late 1920s were so unstable.

Anyone who has walked on an icy sidewalk knows that it can be an unstable proposition. You are likely to experience a very hard fall if you mis-step or have your arm grabbed by someone who is falling themselves. Walking on a dry sidewalk is quite stable because you're much more likely to remain upright and on your feet following the "disturbance" of being bumped or of taking a slight mis-step.

Whether the "system" in our examples consists of the economy or a person walking on a sidewalk, we consider them **stable because they return to their original state of equilibrium following brief disturbances** (the walker stays upright when bumped). Conversely, they are unstable if they settle into a new state following a disturbance (the walker lands on his backside).

Stable vs. Unstable Machine Rotors

Similarly, the conditions in which rotor systems operate cause them to be either stable or unstable. Rotor instability stems from the fluids within fluid film bearings, seals and other close clearance annular spaces surrounding the rotor that flow with sufficient circumferential velocity to cancel the stabilizing damping forces. Known as fluid-induced instabilities, oil whip and oil whirl in fluid film bearings are the most common but certainly not the only forms of rotor instability.

Usually, machine rotors operate in a stable manner and only experience vibration that is "normal" because it is low enough to be considered harmless. The vibration level in stable rotor systems may momentarily increase when the rotor is disturbed but it will quickly return to the harmlessly low level that it had prior to the "bump".

The total amount of vibration that occurs in a machine is really the sum of many individual vibrations that have their own amplitude and frequency. For purposes of understanding rotor stability, this fact allows us to think of the total vibration level as being the sum of two separate parts. Figure 1 illustrates these two parts with the blue curve representing the "normal" *steady state* vibration that is always present and the red curve representing the *transient* vibration that results from a brief disturbance and then dies away.

Steady state vibration results from forces that repeat themselves at regular periodic intervals when conditions in the system are held constant. You may have experienced steady state vibration when driving on a dirt road with a bad washboard. The ripples that create the washboard are evenly spaced with about the same height so that they cause your car to vibrate in a very repeatable manner. Likewise, rotating machines experience steady state vibration due to residual unbalance that exerts a consistent, ongoing centrifugal force on the rotor when it is operated at a constant speed. Transient vibration results from forces that briefly appear and then disappear. Hitting a pothole with your car is (usually) a one-time event that causes transient vibration which may be quite strong at first but then quickly die down. Machine rotors also experience transient vibration from

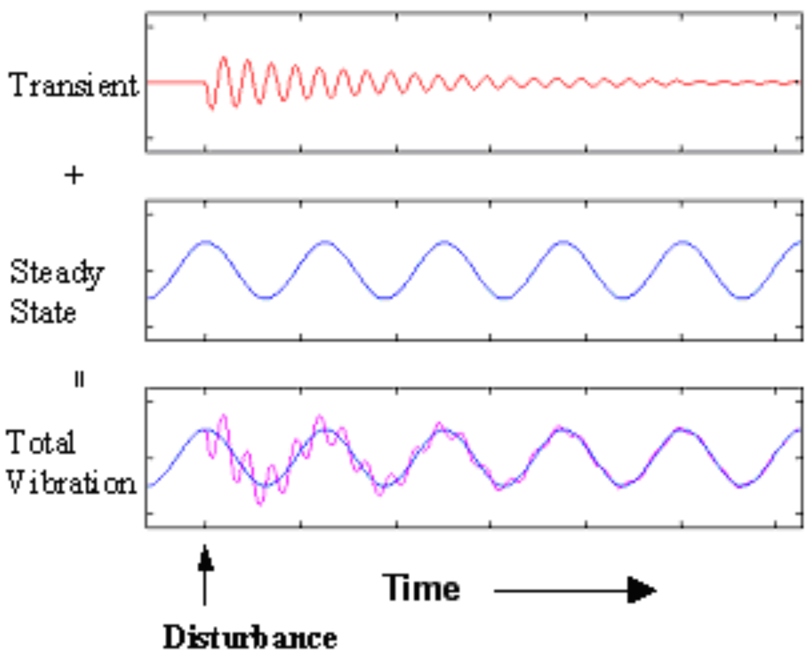

Figure 2: Overall vibration response of a stable rotor system where the transient vibration response decays.

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a multitude of sources in their surrounding environment (aerodynamic forces, other plant processes, etc.).

Prior to the disturbance in Figure 1, the total vibration response only consists of the steady state component. This is also true after the transient response has decayed away. The rotor system is stable because that part of the vibration response which results from the disturbance, the transient response, decays and permits the rotor to continue vibrating at its original steady state level.

The opposite is true with an unstable rotor system. Figure 2 shows that the **transient response of an unstable rotor system will *grow* following a**

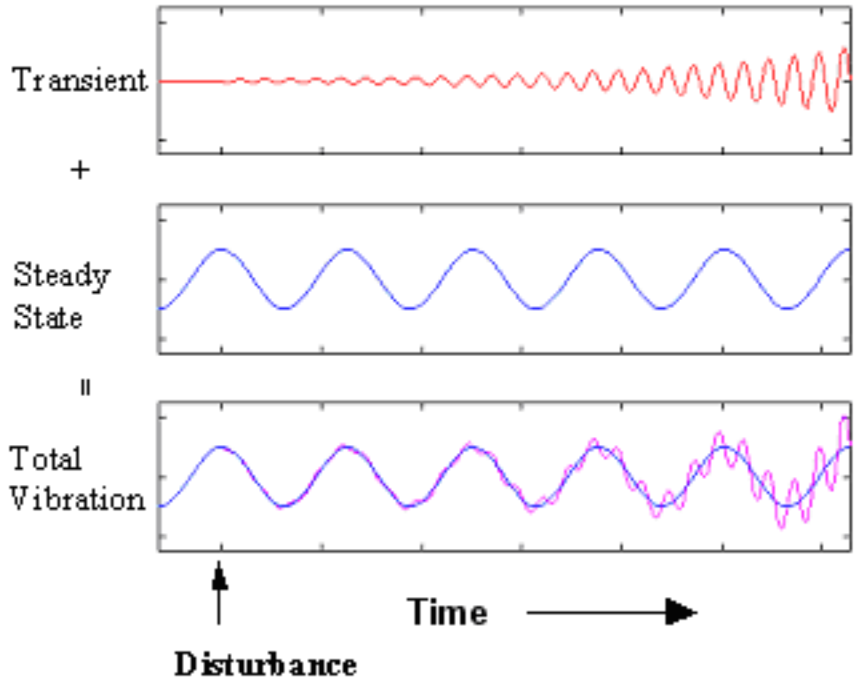


Figure 2

disturbance of the rotor system. Although the steady state response remains unaffected by the disturbance, the combined effect of the unchanging steady state and the increasing transient may cause the total vibration to grow to levels that eventually damage the machine by causing unwanted rubs at bearings, seals, blade tips, etc.

The Practical Side of Root Locus

The real power of any analytical tool is its ability to help us proactively avoid the conditions that lead to instability or, if the damage has already been done, to help us prevent future reoccurrences of the instability. Just as stock market analysts use economic tools to discern market stability and advise their clients, **root locus enables us to analyze rotordynamic stability and recommend solutions that will enable our customers to continue to operate their machines in a stable manner.**

Machine operators and designers are continually pressed to increase output without increasing equipment costs. This leads to the question "How much can I safely increase output by increasing rotor speed without damaging the machine or degrading its performance?" This question has a direct bearing on rotor stability because the transient response in all rotating machines with fluid film bearings will grow instead of decay when the machine is operated above the speed known as the Threshold of Instability. The growing transient response is a fluid-induced instability that can result in high subsynchronous vibration.

Root locus is a powerful analytical tool that can be used to evaluate the Threshold of Instability. Root locus allows "what-if" evaluations of different combinations of the rotor parameters that determine stability. The goal of a typical root locus analysis would be to determine which solution will achieve the highest possible Threshold of Instability at the most economical cost.

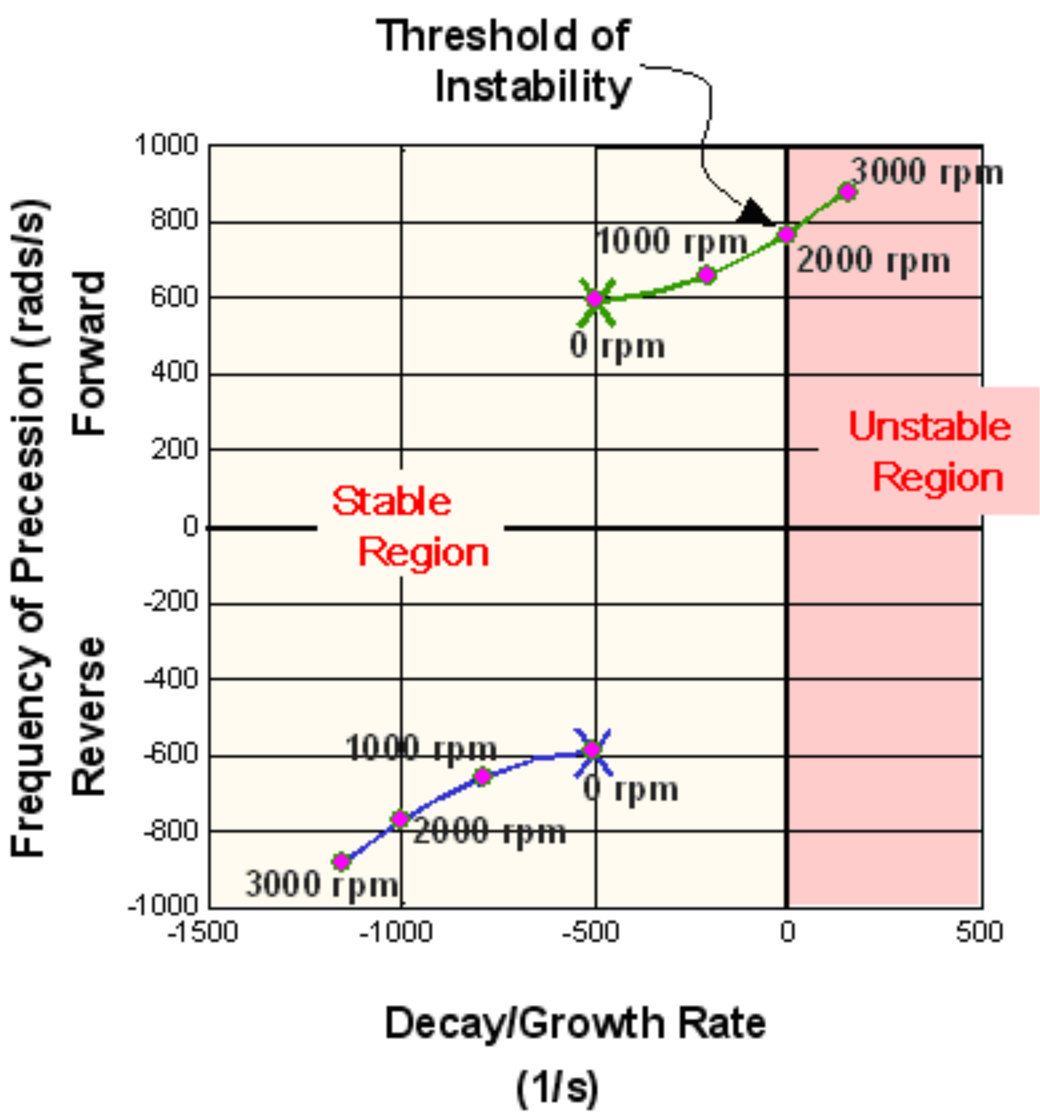
Root Locus Unveiled: Just The Basics

Rotating machines behave as closed-loop control systems where the vibration response depends upon the impinging forces and moments. This fact permits them to be analyzed using control theory techniques including one known as root locus. Root locus is a method of stability and transient response analysis that was developed by W. R. Evans in 1948[2]. Since rotor stability analysis must focus on the stability of the transient response, root locus becomes an ideal method of choice because it graphically distinguishes stable rotor conditions from those that produce instability. In addition, the root locus plot preserves information about the frequency of the unstable rotor motion. This gives it a distinct advantage over other stability analysis tools because it allows for the derivation of several other key pieces of information about the rotor system.

Root locus plots are XY plots (Figure 3, top) that are divided horizontally and vertically about the horizontal and vertical 0 axes. The left side of the plot represents transient vibration that will decay with time hence making this the stable plot region. Conversely, the right side of the plot represents transient vibration that grows therefore making it the unstable plot region. Figures of stable and unstable motion are included below the root locus plot to graphically emphasize this point, they are not normally included as part of a root locus plot. More on the interpretation of these figures follows below.

The top half of the root locus plot represents *forward* transient rotor motion that vibrates, or precesses, in the same direction as shaft rotation. The bottom half represents *reverse* rotor vibration that precesses in the direction opposite to shaft rotation. All orbital rotor motion is a combination of various degrees of forward and reverse motions. Orbits that are purely circular in the same direction as shaft rotation are comprised entirely of forward vibration components. Orbits become less circular and more elliptical as the reverse component increases in magnitude. Moving from right to left across the bottom of Figure 3, observe how the actual rotor paths become less circular as their reverse components increase. More on the subject of forward and reverse vibration is available in resources listed at the end of this article [1].

Since rotor motion requires both a forward and reverse component in order to be completely described, **the points on a root locus plot always occur in pairs**. In the root locus plot in Figure 3, notice that each point on the top branch (green) has a corresponding point on the bottom branch (blue). Summing the forward and reverse component at each speed results in the actual rotor paths shown at the bottom of Figure 3.



Root Locus Plot (top)

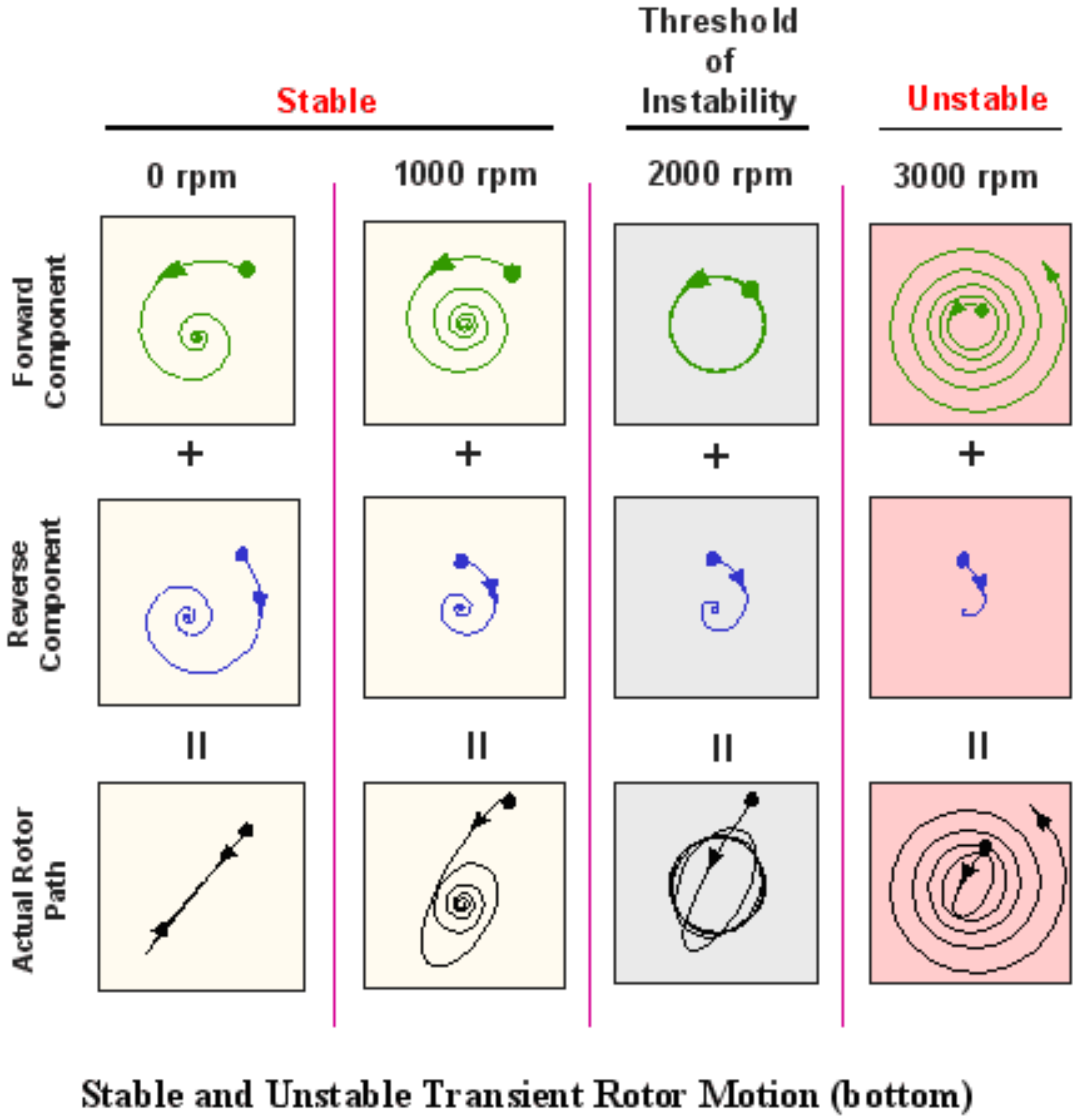


Figure 3

Moving up the vertical axis indicates increasingly faster forward precession frequency while moving down the vertical axis indicates increasingly faster reverse precession frequency. A root locus point that has zero precession frequency would be plotted directly on the horizontal axis. Zero precession frequency means the rotor would not vibrate when disturbed but simply return to its original equilibrium position in the shortest possible amount of time.

The bottom half of Figure 3 shows four columns of rotor motion. These figures illustrate what would happen if we could disturb the rotor by pulling it away from its equilibrium position and then release it, much as occurs when a guitar string is plucked. Stable motion allows the rotor to return to its original equilibrium position along the actual rotor path after several cycles of vibration. Unstable motion spirals outward along the actual rotor path until it comes to rest in a new equilibrium. The new equilibrium occurs when the rotor meets with some constraining feature within the machine. Machine damage results when the bearing, seal or other feature that now constrains the rotor is incapable of carrying the undesired rotor-to-stator contact.

Recall that root locus analysis allows "what-if" studies of rotor stability. These studies are conducted by developing a mathematical model of the rotor system and then solving it while varying one of the parameters with the rest held constant. In the example shown in Figure 3, rotor speed was varied while other parameters such as mass, stiffness and damping were held constant. Solving the model in this manner generates different points as rotor speed is varied. **Each pair of points on the root locus plots represents the decay rate and precession frequency of the transient response for a different set of rotor parameters.** The actual rotor paths at the bottom of Figure 3 show the length of time and the path followed by each response as they either decay or grow.

We can develop a better intuitive feel for what a root locus plots tells us if we understand how the decay/growth rate changes along the horizontal axis. The transient response decays more quickly as it plots further to the left and it grows more quickly as it plots further to the right. Looking at either the forward or reverse components at the bottom of Figure 3 helps to illustrate this. Notice that the 0 rpm forward component decays to a point after only a few cycles of vibration while the 1000 rpm forward component does not decay to a point until after about twice as many vibration cycles.

The 2000 rpm forward component falls directly on the vertical axis and is purely circular because it neither decays nor grows. This marks the Threshold of Instability because the transient response can no longer decay on or to the right of the vertical axis. At 3000 rpm the response grows with time.

The reverse components also show that decay rate increases as they plot further to the left on the root locus plot. The reverse components at the bottom of the Figure 3 show that their decay rate is increasingly rapidly as rotor speed increases from 0 to 3000 rpm. The reverse component decays so rapidly at 3000 rpm that there is not sufficient time for even a single cycle of vibration. It is interesting to observe that the 0 rpm forward and reverse root locus points are directly above and below each other and that their motions are mirror images of each other. These points represent the transient response of a stopped rotor which, simplistically speaking, does vibrate in a straight line like a giant guitar string. Since the forward and reverse components are of equal decay rate, they effectively cancel each other leaving a straight line motion.

The dynamic forces in a rotor system cause the forward component to decay more slowly and the reverse component to decay more rapidly as speed increases. This results in forward root locus branches that typically move to the right with increasing speed and reverse branches that move to the left, as shown in Figure 3. Consequently, the forward branch usually gets the most attention during a root locus analysis because it will cross the vertical axis at the Threshold of Instability.

More to Follow: In-Depth Root Locus

Hopefully, this article has implanted something of an intuitive feel for root locus analysis. At its most basic level, **root locus indicates stability of the transient rotor response in terms of its decay/growth rate *and* its frequency.**

Space permits that we can only address introductory concepts here. Root locus analysis can tell us much more about the rotor system including: the synchronous resonance speed, rotor system damping factor and the synchronous amplification factor (SAF). Also, it is important to understand the techniques used to generate root locus plots so that the information in them can be applied in an appropriate manner.

These and other concepts will be the subject of future articles on root locus. Extensive information on root locus as a rotor stability tool is available in additional resources [1].

References:

1. Bently, Donald E., “Fundamentals of Rotating Machinery Diagnostics”, First Edition, Bently Pressurized Bearing Press, 2002

2. Evans, Walter R., “Control-System Dynamics”, McGraw-Hill, 1954