The Death of Whirl –

What the SFCB Can Do for the Stability of Rotating Machinery

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Our new ServoFluid™ Control Bearing (SFCB) eliminates fluid whirl. Whip, another common instability, can only be partially addressed by the SFCB. However, the bearing still offers improvement over conventional technologies in treating this type of instability and may be a viable solution, depending on the application. While we've physically tested and demonstrated the bearing's effect on whirl and whip in machines ranging from full-sized compressors to rotor kits, this article provides the technical and mathematical rigor to substantiate our claims regarding the stability of this pressurized bearing technology. The equations related to stability used in this article were originally jointly developed by Don Bently and Dr. Agnes Muszynska.

Two principles of SFCB operation

The best way to understand how the SFCB works is to first review how conventional hydrodynamic and hydrostatic bearings work. Our bearing exhibits certain characteristics of both bearing types, but combines them in an innovative way that gives new and significant improvements over these other designs.

Fluid-film bearings have historically been classified as one of two types:

- Hydrodynamic
- Hydrostatic

As these names imply, one bearing – the hydrodynamic type – relies on dynamic (motion) principles, as a means of developing the fluid film between the bearing and its journal. The other bearing type – hydrostatic – relies on static (no motion) principles where external pressurization of the lubricant is used to develop a fluid film. The SFCB combines the effects of both types of bearings.

Hydrodynamic Bearings

The stiffness and damping characteristics of these bearings [1] are mathematically described by the following equations:

$$K_{BD} = \frac{\eta \Omega d l^3}{c^3} \frac{K_1 \varepsilon}{\left(1-\varepsilon^2\right)^{5/2}}, \left(N/m, lb/in\right) \qquad (1)$$

$$D_{BD} = \frac{\eta d l^3}{c^3} \frac{D_1}{\left(1-\varepsilon^2\right)^{3/2}}, \left(\frac{N \cdot \sec}{m}, \frac{lb \cdot \sec}{in}\right) \qquad (2)$$

Where:

K_{BD}, D_{BD}	are radial stiffness and damping of the hydrodynamic bearing, respectively
K_1, D_1	are constants
d, c	are journal diameter and diametral clearance, respectively
l	is bearing length
η	is dynamic viscosity
ε	is eccentricity ratio
Ω	is rotative speed

As can be seen, these two bearing parameters (stiffness and damping) are coupled to one another. In other words, attempts to change the damping result in changes to the stiffness and vice-versa. For example, equations (1) and (2) show that an attempt to raise the stiffness by increasing fluid viscosity, η, will also result in an increase in damping, since both equations are directly proportional to η. If a designer chooses a "perfect" stiffness for the bearing's application, this may result in non-optimal damping, and vice-versa. This fundamental constraint means that bearing designers must often pursue compromises, rather than optimal stiffness and damping for a particular application. As we will see, this is not a problem for the ServoFluid™ Control Bearing.

Also, note that stiffness is directly proportional to the shaft's rotative speed, Ω. This is intuitive in a hydrodynamic bearing because the dynamic motion (between the bearing and journal surfaces) is what develops the supporting wedge.

Stiffness of the SFCB as a combination of hydrodynamic and hydrostatic effects

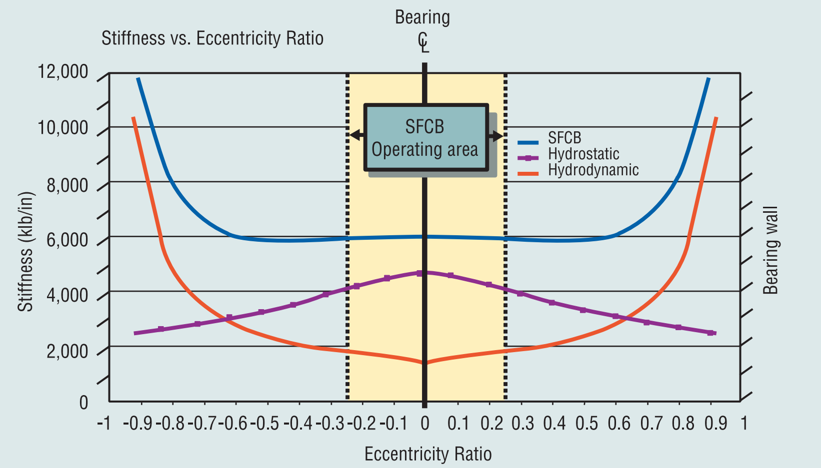

Figure 1.

Hydrostatic Bearings

The stiffness characteristic of these bearings is mathematically described by the following equations from Rowe and O'Donoghue [2]:

$$K_{BS} = \frac{2P_s}{cl}\frac{d}{l-a}C_0 \tag{3}$$

Where:

K_{BS}	=	stiffness of hydrostatic bearing
P_s	=	supply pressure
d	=	bearing diameter
c	=	bearing to journal diametral clearance
l	=	length of bearing
a	=	length of a single axial land
C_0	=	non-dimensional stiffness based on bearing geometry (γ) and pressures (β)

$$C_0 = \frac{7.65\,\beta(1-\beta)}{2-\beta+\gamma(1-\beta)} \sim 1 \tag{4}$$

$$\beta = \frac{P_p}{P_s} \sim 0.5 \tag{5}$$

$$\gamma = \frac{na(l-a)}{\pi db} \sim 0.5 \tag{6}$$

Where:

P_p	=	Pocket pressure
n	=	number of pockets
b	=	length of single circumferential land

The damping of a hydrostatic bearing is described by the same equation (2) as for a hydrodynamic bearing. Notice in equation (3) that the stiffness in this type of bearing is related to P_s, the differential pressure of the lubricant across the supply and drain ports of the bearing. This is intuitive as well. We would expect that as we increase the pressure of the lubricant, we could make the bearing stiffer.

ServoFluid™ Control Bearing

The ServoFluid™ Control Bearing combines the stiffness of hydrodynamic and hydrostatic bearings.

The resulting stiffness is as follows:

$$K_B = K_{BS} + K_{BD} \tag{7}$$

while damping is the same as in a hydrostatic bearing.

Note that the SFCB stiffness characteristics have a significant stiffness at low eccentricities as in a hydrostatic bearing, and steeply increasing stiffness close to the wall, as in a hydrodynamic bearing (Figure 1). Since the hydrostatic portion of the stiffness of the SFCB is independent of the fluid viscosity, changes in fluid viscosity can be used as a means to adjust damping and thereby optimize the damping-to-stiffness ratio.

Whirl and Whip

In order to describe the influence of the bearing characteristics on the fluid-induced whirl/whip instability of a rotor/bearing system, a simple rotor is considered (Figure 2).

The parameters of this system include mass of the rotor M, stiffness of the shaft K_s, stiffness K_B, damping D_B, and average circumferential velocity ratio λ of the fluid-film bearing. The rotor lateral response is described by the vector of displacement $\boldsymbol{r} = x + jy$ (x and y are vertical and horizontal displacements, correspondingly, and $j = \sqrt{-1}$). The vector of shaft bending, $\boldsymbol{r_1}$, and the vector of the journal displacement, $\boldsymbol{r_2}$, constitute the vector $\boldsymbol{r}$. The equations describing the motion of this system are as follows:

$$\begin{aligned} &M\ddot{\boldsymbol{r}} + K_s\left(\boldsymbol{r} - \boldsymbol{r_2}\right) = Ma\Omega^2 e^{j(\Omega t + \alpha)} \\ &K_s\boldsymbol{r_1} = D_B\dot{\boldsymbol{r}}_2 + \left(K_B - jD_B\lambda\Omega\right)\boldsymbol{r_2} \\ &\boldsymbol{r} = \boldsymbol{r_1} + \boldsymbol{r_2} \end{aligned} \tag{8}$$

Here a and α are radial and angular coordinates of the rotor center of mass. The lowest possible rotative speed Ω_{th} at which the synchronous solution of the system (8) becomes unstable, is described by the Bently-Muszynska instability threshold [3]:

$$\Omega_{th} = \frac{1}{\lambda}\sqrt{\frac{K_s K_{B0}}{M\left(K_s + K_{B0}\right)}} \tag{9}$$

where K_{B0} is a fluid-film bearing stiffness at the journal central position (zero eccentricity). According to expression (7), this stiffness is determined only by the hydrostatic component:

$$K_{B0} = K_{BS} = \frac{2P_s}{cl}\frac{d}{l-a}C_0 \tag{10}$$

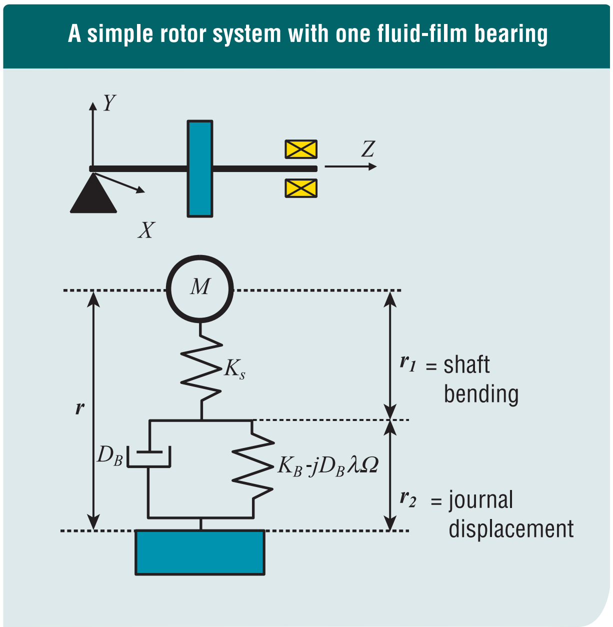

Figure 2.

and is proportional to the supply pressure. As long as the bearing stiffness is lower than the stiffness of the shaft, it controls the instability threshold. In that case, an increase in the bearing's supply pressure can move the instability threshold above the operational speed. However, when the bearing stiffness becomes higher than the stiffness of the shaft, the increase in the bearing stiffness does not significantly increase the instability threshold.

After the synchronous solution becomes unstable, the system experiences self-excited vibration, which can be described by the same set of equations, but with unbalance forces neglected. The solution can be found in the form of harmonic oscillations with unknown frequency of self-excited vibration, ω. This frequency satisfies the characteristic equation:

$$\begin{vmatrix} K_s & -K_B - K_s - jD_B\left(\omega - \lambda\Omega\right) \\ K_s - M\omega^2 & -K_s \end{vmatrix} = 0 \tag{11}$$

Direct and Quadrature parts of this equation are as follows:

$$\frac{K_s K_B}{M\left(K_s + K_B\right)} - \omega^2 = 0 \qquad \text{Direct} \tag{12a}$$

$$\left(K_s - M\omega^2\right)\left(\omega - \lambda\Omega\right) = 0 \qquad \text{Quadrature} \tag{12b}$$

Equations 12a and 12b are simultaneously satisfied when:

$$\omega = \lambda\Omega = \sqrt{\frac{K_s K_B}{M(K_s K_B)}} = \sqrt{\frac{1}{M}\left[\frac{K_s}{(K_s / K_B)+1}\right]} \quad (13a)$$

or, when:

$$\omega = \sqrt{\frac{K_s}{M}} = \sqrt{\frac{1}{M}\,\frac{K_s K_B}{(K_s + K_B)}} \quad (13b)$$

These solutions lead to two distinct types of instability phenomena. The first is known as fluid whirl and occurs when the conditions of equation (13a) are met. When whirl occurs, the bearing stiffness has primary influence on the instability response. The second type of instability occurs when the conditions of equation (13b) are met. This leads to a whip instability. However, an examination of equation (13b) will show that it can only hold as $K_B \rightarrow \infty$. In practice, we never really have an infinitely stiff bearing. Instead, we see a whip instability emerge as $K_B >> K_S$. Whip occurs when the shaft stiffness provides the primary influence over the instability response.

1. **Whirl**

As noted above, whirl occurs when the conditions of equation (13a) are met. In the whirl regime, the amplitude of the journal displacement, r_2, is much higher than that of the shaft bending, r_1, while the phases are essentially the same. This indicates a predominantly rigid-body mode of response, typical for whirl. In other words, the bearing stiffness tends to dominate the instability response.

By increasing the bearing stiffness K_B in equation (9), we can arbitrarily raise the instability threshold Ω_{th}. ***This is precisely why the SFCB eliminates whirl problems – because bearing stiffness K_B can be increased independent of damping to raise the threshold of instability above operating rotative speeds, precluding whirl from ever occurring.*** The bearing is designed initially with proper stiffness to locate the instability threshold above operating speeds. If conditions change for some reason, the bearing stiffness can be raised in the field by raising the fluid supply pressure, further increasing the threshold of stability.

2. **Whip**

We have already noted that when the conditions of equation (13b) are met, we see a different kind of instability response, known as whip. We have also noted that although our bearing can never really become infinitely stiff, its stiffness can greatly exceed that of the shaft. Essentially, what equation (13b) tells us is that there is a point at which increases in bearing stiffness no longer influence the instability threshold; it is influenced instead by the shaft stiffness. This is intuitive. As the bearing stiffness becomes considerably larger than the shaft stiffness, the least stiff spring in the system (the shaft stiffness) begins to dominate the response and further increasing the bearing stiffness no longer influences the response. Figure 2 helps to make this clear as well. Notice that we effectively have two springs in series – one governed by the shaft stiffness K_S and one governed by the bearing stiffness K_B. If the "bearing" spring (the one influenced by K_B) is replaced with an infinitely stiff spring (i.e., a straight line), the system still has a spring, influenced by K_S. This is what is shown in equation (13b).

This region in which shaft stiffness dominates the instability response is known as whip. As we have shown, raising the bearing stiffness further will not necessarily control whip problems. Instead, the shaft stiffness must be addressed, or λ must be reduced, or both. ***Thus, whip, as a predominantly rotor-bending mode instability response, can only be partially controlled by the SFCB.***

The SFCB can help with whip problems in the following ways:

- Axial fluid flow in the SFCB can be increased to reduce λ.
- It may be possible to install a SFCB in place of an existing mid-span seal to raise the shaft stiffness K_S.

Also, anti-swirl devices can be installed to reduce λ.

Summary

This article showed the principles of operation for both hydrodynamic and hydrostatic bearing types. It was further shown that the SFCB exhibits characteristics of both bearing types. Its stiffness is simply a linear summation of the hydrostatic and hydrodynamic stiffnesses, and the damping is the same as in a hydrostatic bearing. Mathematically and using intuitive explanations, the ability to totally eliminate whirl instabilities was demonstrated, and the special considerations for partially controlling whip instabilities was shown.

References:

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